ON-LINE ESTIMATION OF THE CORE LOSS RESISTANCE AND THE EDDY-CURRENT-ASSOCIATED LEAKAGE INDUCTANCE IN THE PARALLEL MODEL OF THE INDUCTION MOTOR

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Abstract: This article presents a new online method for estimating the core-loss resistance and the leakage inductance associated with eddy currents in the parallel model of the induction motor. The proposed method uses information regarding the stator current, stator voltage, and motor rotational speed to identify additional parameters of the equivalent circuit, contributing to a more accurate description of the machine's electromagnetic behavior. The performance of the estimator is evaluated through numerical simulation performed in the Matlab-Simulink environment.

Key words: induction machines, simulation, core-loss resistance, leakage inductance.

1. INTRODUCTION

Induction motors represent the backbone of modern industrial systems due to their robustness, efficiency, and low manufacturing cost. Achieving high-performance operation of these machines, especially within vector-control frameworks, requires an accurate mathematical representation of their electrical and magnetic behavior. This accuracy critically depends on the precise knowledge of the motor's electrical parameters [2], [6], [7], [10].

While classical induction motor models typically neglect iron losses, numerous studies have demonstrated that these losses significantly influence both transient and steady-state behavior, especially when the motor is supplied by voltage-source inverters operating at high switching frequencies [4], [5]. To address this limitation, the parallel model of the induction motor includes a dedicated core-loss branch consisting of two key parameters: the core-loss resistance (R_f), representing hysteresis and resistive eddy-current losses, and the eddy-current–associated leakage inductance (L_f), modeling the dynamic behavior of eddy currents in the stator core [3], [4], [5].

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Accurate identification or estimation of R_f and L_f is essential for improving the fidelity of the mathematical model, for enhancing the accuracy of flux observers, and for achieving superior dynamic performance in vector control systems. However, these parameters are highly dependent on excitation frequency, magnetic saturation, and temperature, making their estimation nontrivial. Despite their importance, only a limited number of studies have proposed practical and reliable estimation procedures for the parameters of the iron-loss branch in the parallel model.

Given this context, this article proposes an on-line estimation method for determining the core-loss resistance R_f and the eddy-current-associated leakage inductance L_f of the induction motor.

2. THE MATHEMATICAL MODELS OF THE INDUCTION MOTOR

The equations that define the parallel mathematical model of the induction motor, written based on the equivalent circuit (see Fig. 1), in a stator reference frame, are presented below [9].

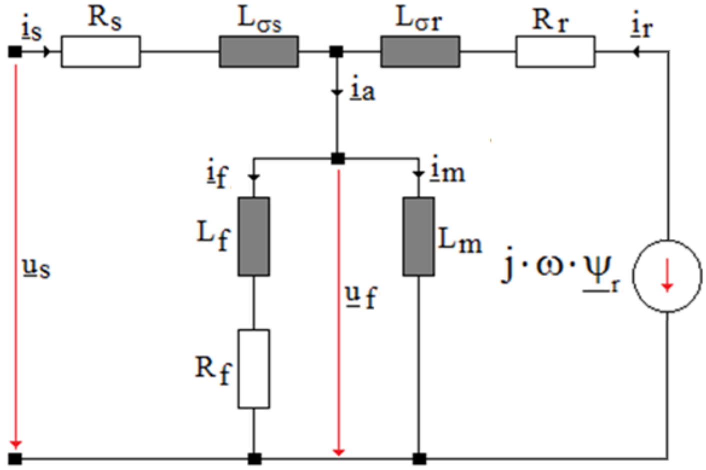

Fig.1. The equivalent circuit of the induction motor

- the stator-voltages equation

$$\underline{u}_s = R_s \cdot \underline{i}_s + \frac{d}{dt}\underline{\psi}_s \tag{1}$$

- the rotor-voltages equation

$$0 = R_r \cdot \underline{i}_r + \frac{d}{dt}\underline{\psi}_r - j \cdot \omega \cdot \underline{\psi}_r \tag{2}$$

- the stator flux equation

$$\underline{\psi}_s = \underline{\psi}_m + L_{\sigma s} \cdot \underline{i}_s \tag{3}$$

- the rotor flux equation

$$\underline{\psi}_r = \underline{\psi}_m + L_{\sigma r} \cdot \underline{i}_r \tag{4}$$

- the air-gap flux equation

$$\underline{\psi}_m = L_m \cdot \underline{i}_m \tag{5}$$

- the current equation

$$\underline{i}_s + \underline{i}_r = \underline{i}_m + \underline{i}_f \tag{6}$$

- magnetic branch voltage equation

$$\underline{u}_f = R_f \cdot \underline{i}_f + L_f \cdot \frac{d}{dt}\underline{i}_f = \frac{d}{dt}\underline{\psi}_m \tag{7}$$

- the motion equation of the induction motor

$$J \cdot \frac{d}{dt}\omega_r = T_e - F \cdot \omega_r - T_L \tag{8}$$

where T_e is the electromagnetic torque and T_L is the load torque

$$T_e = \frac{3}{2} \cdot \frac{z_p}{L_{\sigma r}} \cdot \mathrm{Im}\left(\underline{\psi}_r^* \cdot \underline{\psi}_m\right) \tag{9}$$

The following notations were used in the above relationships:

$\underline{u}_s = u_{ds} + j \cdot u_{qs}$; $\underline{u}_f = u_{df} + j \cdot u_{qf}$; $\underline{i}_s = i_{ds} + j \cdot i_{qs}$; $\underline{i}_r = i_{dr} + j \cdot i_{qr}$; $\underline{i}_m = i_{dm} + j \cdot i_{qm}$; $\underline{i}_f = i_{df} + j \cdot i_{qf}$; $\underline{\psi}_s = \psi_{ds} + j \cdot \psi_{qs}$; $\underline{\psi}_r = \psi_{dr} + j \cdot \psi_{qr}$; $\underline{\psi}_r^* = \psi_{dr} - j \cdot \psi_{qr}$; $\underline{\psi}_m = \psi_{dm} + j \cdot \psi_{qm}$; $j = \sqrt{-1}$; $\omega = z_p \cdot \omega_r$; $L_r = L_{\sigma r} + L_m$; $L_s = L_{\sigma s} + L_m$.

3. ON-LINE ESTIMATION OF R_f AND L_f PARAMETERS

The method for estimating the parameters R_f and L_f is based on relation (7). Relation (7) can be written as follows

$$\begin{bmatrix} u_{df} \\ u_{qf} \end{bmatrix} = A \cdot \begin{bmatrix} R_f \\ L_f \end{bmatrix} \tag{10}$$

where: $A = \begin{bmatrix} i_{df} & \frac{d}{dt} i_{df} \\ i_{qf} & \frac{d}{dt} i_{qf} \end{bmatrix}$.

Using the least-squares method, we can extract the parameters R_f and L_f (the system solution), as follows

$$\begin{bmatrix} \hat{R}_f \\ \hat{L}_f \end{bmatrix} = \left(B^T \cdot B \right)^{-1} \cdot B^T \cdot \begin{bmatrix} \hat{u}_{df} \\ \hat{u}_{qf} \end{bmatrix} \tag{11}$$

where: $B = \begin{bmatrix} \hat{i}_{df} & \frac{d}{dt} \hat{i}_{df} \\ \hat{i}_{qf} & \frac{d}{dt} \hat{i}_{qf} \end{bmatrix}$; $\hat{i}_{df}$, $\hat{i}_{qf}$, $\hat{u}_{df}$ and $\hat{u}_{qf}$ are the iron loss currents and iron branch voltages estimated.

Considering relations (1), (3), and (7), the following expression can be written:

$$\underline{u}_f = \underline{u}_s - R_s \cdot \underline{i}_s - L_{\sigma s} \cdot \frac{d}{dt} \underline{i}_s \tag{12}$$

Under these conditions, $\hat{u}_{df}$ and $\hat{u}_{qf}$ can be estimated as follows

$$\begin{bmatrix} \hat{u}_{df} \\ \hat{u}_{qf} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -R_s & 0 & -L_{\sigma s} & 0 \\ 0 & 1 & 0 & -R_s & 0 & -L_{\sigma s} \end{bmatrix} \cdot \begin{bmatrix} u_{ds} \\ u_{qs} \\ i_{ds} \\ i_{qs} \\ \frac{d}{dt} i_{ds} \\ \frac{d}{dt} i_{qs} \end{bmatrix} \tag{13}$$

where u_{ds} and u_{qs}, respectively i_{ds} and i_{qs}, are the stator voltages and currents in the d-q reference frame, obtained by applying the Clarke transformation to the voltages and currents measured, using the voltage and current transducers.

On the other hand, taking into account relations (6), (5) and (4) we can write the following expression:

$$\underline{i}_f = \underline{i}_s + \frac{1}{L_{\sigma r}} \cdot \underline{\psi}_r - \left(\frac{1}{L_{\sigma r}} + \frac{1}{L_m} \right) \cdot \underline{\psi}_m \tag{14}$$

Under these conditions, $\hat{i}_{df}$ and $\hat{i}_{qf}$ can be estimated as follows

$$\begin{bmatrix}\hat{i}_{df}\\ \hat{i}_{qf}\end{bmatrix}=\begin{bmatrix}1 & 0 & \frac{1}{L_{\sigma r}} & 0 & -\left(\frac{1}{L_{\sigma r}}+\frac{1}{L_m}\right) & 0\\ 0 & 1 & 0 & \frac{1}{L_{\sigma r}} & 0 & -\left(\frac{1}{L_{\sigma r}}+\frac{1}{L_m}\right)\end{bmatrix}\cdot\begin{bmatrix}i_{ds}\\ i_{qs}\\ \hat{\psi}_{dr}\\ \hat{\psi}_{qr}\\ \hat{\psi}_{dm}\\ \hat{\psi}_{qm}\end{bmatrix} \tag{15}$$

where i_{ds} and i_{qs}, are the stator currents in the d-q reference frame, obtained by applying the Clarke transformation to the currents measured, using the current transducers.

In relation (15), $\hat{\psi}_{dr}$, $\hat{\psi}_{qr}$, and respectively $\hat{\psi}_{dm}$ and $\hat{\psi}_{qm}$ are the d–q components of the estimated rotor flux and of the estimated magnetizing flux.

In order to estimate the magnetization flux, relations (1) and (3) are used. Thus, the relations that define this estimator are

$$\underline{\hat{\psi}}_m=\underline{\hat{\psi}}_s-L_{\sigma s}\cdot\underline{i}_s \tag{16}$$

where $\underline{\hat{\psi}}_s=\int_0^t\left(\underline{u}_s-R_s\cdot\underline{i}_s\right)d\tau$.

Relation (16) can also be written in the following form

$$\begin{bmatrix}\hat{\psi}_{dm}\\ \hat{\psi}_{qm}\end{bmatrix}=\begin{bmatrix}1 & 0 & -L_{\sigma s} & 0\\ 0 & 1 & 0 & -L_{\sigma s}\end{bmatrix}\cdot\begin{bmatrix}\hat{\psi}_{ds}\\ \hat{\psi}_{qs}\\ i_{ds}\\ i_{qs}\end{bmatrix} \tag{17}$$

where: $\hat{\psi}_{ds}=\int_0^t\left(u_{ds}-R_s\cdot i_{ds}\right)d\tau$; $\hat{\psi}_{qs}=\int_0^t\left(u_{qs}-R_s\cdot i_{qs}\right)d\tau$.

On the other hand, taking into account relations (2) and (4) the estimator related to the rotor flux is defined by the following expression.

$$\frac{d}{dt}\underline{\hat{\psi}}_r=\left(-\frac{R_r}{L_{\sigma r}}+j\cdot z_p\cdot\omega_r\right)\cdot\underline{\hat{\psi}}_r+\frac{R_r}{L_{\sigma r}}\cdot\underline{\hat{\psi}}_m \tag{18}$$

Relation (18) can also be written in the following form

$$\begin{bmatrix} \dot{\hat{\psi}}_{dr} \\ \dot{\hat{\psi}}_{qr} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_r}{L_{\sigma r}} & -z_p \cdot \omega_r \\ z_p \cdot \omega_r & -\dfrac{R_r}{L_{\sigma r}} \end{bmatrix} \cdot \begin{bmatrix} \hat{\psi}_{dr} \\ \hat{\psi}_{qr} \end{bmatrix} + \frac{R_r}{L_{\sigma r}} \cdot \begin{bmatrix} \hat{\psi}_{dm} \\ \hat{\psi}_{qm} \end{bmatrix} \tag{19}$$

The on-line estimation method presented above is summarized in the following block diagram (see Fig.2).

In Fig. 2, **IM** denotes the induction motor, $\mathbf{T_\omega}$ denotes the speed transducer, and **TS** (system transformations) denotes the Clarke transformer block.

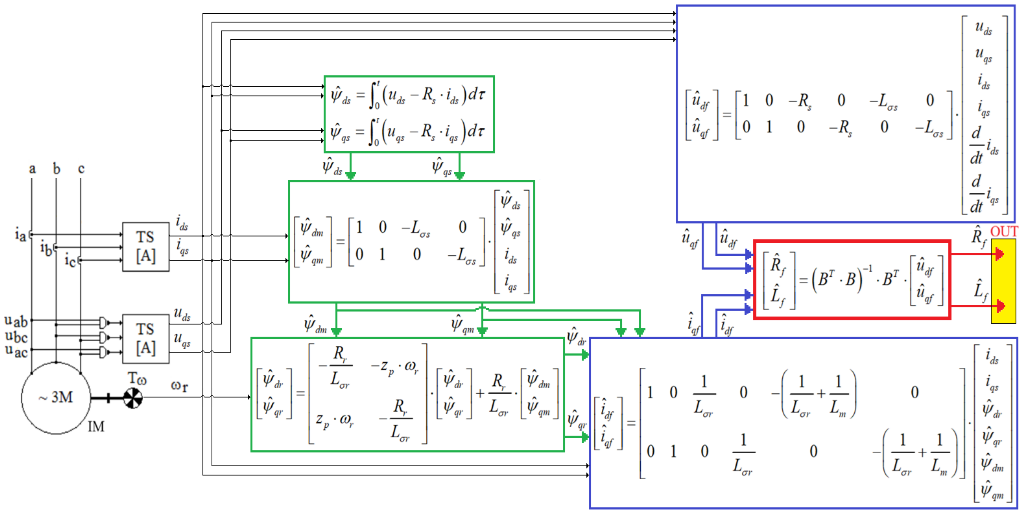

Fig.2. Block diagram of the R_f and L_f on-line estimation method

From the above, it can be seen that the on-line estimation method for R_f and L_f is based on three flux observers (stator flux observer, air-gap flux observer and rotor flux observer).

4. SIMULATIONS RESULTS

To test the previously presented online estimator, a 1.5 [kW] induction motor is used, which has the electrical and mechanical parameters presented in Table 1.

The estimator is tested via simulation in the Matlab-Simulink environment, in the case of direct on-load (DOL) starting of the induction motor. The induction motor is started under load, with a torque equal to its rated value.

Both the induction motor and the estimator presented previously are simulated in continuous time in Matlab-Simulink.

In the simulation test, the ode15s (stiff/NDF) integration method is used, with a relative and absolute error of 10^{-12}. The final simulation time is t_f=1 [s].

In the case of the simulation test, the induction motor is supplied with three-phase voltages at a frequency of 50 [Hz].

Table 1. **The electrical and mechanical parameters of the induction motor [1]**

	Name	Value		Name	Value
R_s	*Stator resistance*	4.85 [Ω]	J	*Motor inertia*	0.031[kg·m^2]
R_r	*Rotor resistance*	3.805 [Ω]	F	*Friction coefficient*	0.008[N·m·s/rad]
R_f	*Core loss resistance*	500 [Ω]	n_N	*Rated speed*	1420 [rpm]
L_s	*Stator inductance*	0.274 [H]	z_p	*Number of pole pairs*	2
L_r	*Rotor inductance*	0.274 [H]	f_N	*Rated frequency*	50 [Hz]
L_m	*Mutual inductance*	0.258 [H]	U_N	*Rated voltage*	220 Δ/380 Y [V]
L_f	*Eddy currents leakage inductance*	0.1 [H]	M_N	*Rated torque*	10 [N·m]

The simulation model of the induction motor – R_fL_f estimator assembly, is presented in Fig. 3.

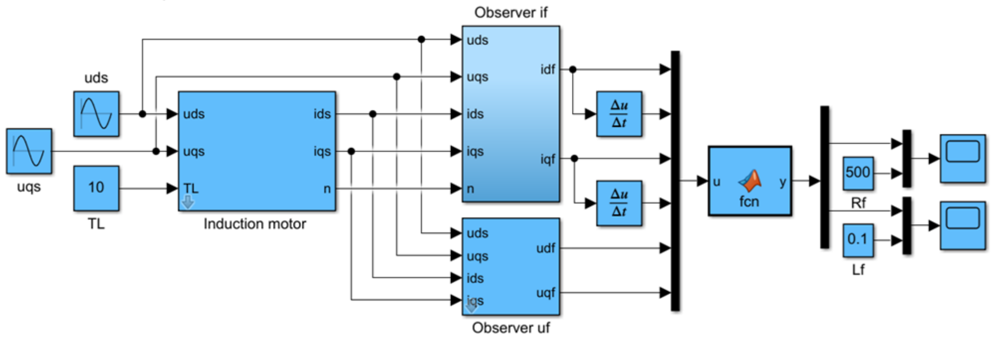

Fig.3. Matlab-Simulink simulation model of the R_fL_f estimator

In the simulation program in Fig.3, the induction motor is simulated in continuous time, using an S-Function block [8], [9].

The internal structure of the block used in estimating the $u_{df} – u_{qf}$ stresses is shown in Fig. 4.

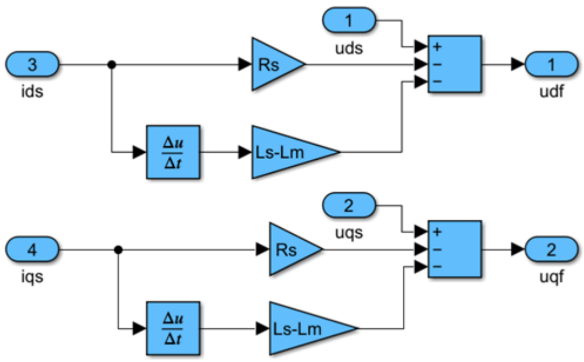

Fig.4. The internal structure of the block used in $u_{df} – u_{qf}$ estimating

On the other hand, the internal structure of the block used in estimating the d-q components of the current $\underline{i}_f$, is presented in Fig.5.

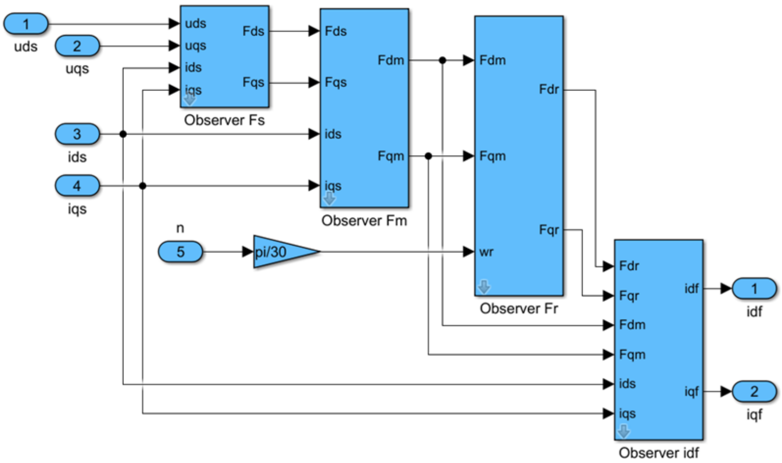

Fig.5. The internal structure of the block used in $i_{df} - i_{qf}$ estimating

The Matlab function used in estimating R_f and L_f, taking into account relation (11), is presented in Fig.6.

```
function y = fcn(u)
Ba=[u(1) u(2);
   u(3) u(4)];
Ua=[u(5);u(6)];
y = (inv(Ba'*Ba))*(Ba')*Ua;
```

Fig.6. The Matlab function used in estimating Rf and Lf

After running the program in Fig.3, under the previously presented conditions, the following results are obtained (see Fig. 7 and Fig. 8).

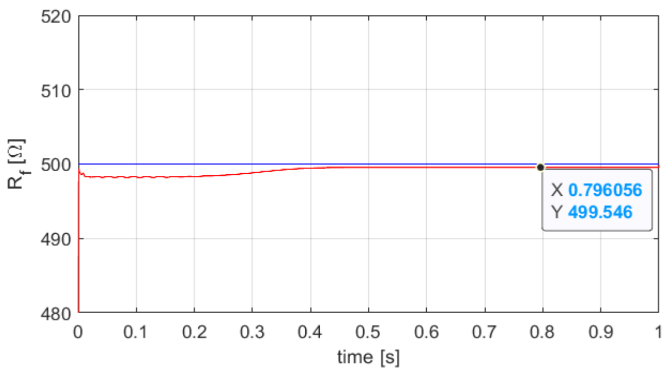

Fig.7. The variation over time of the estimated R_f resistance, together with the reference value.

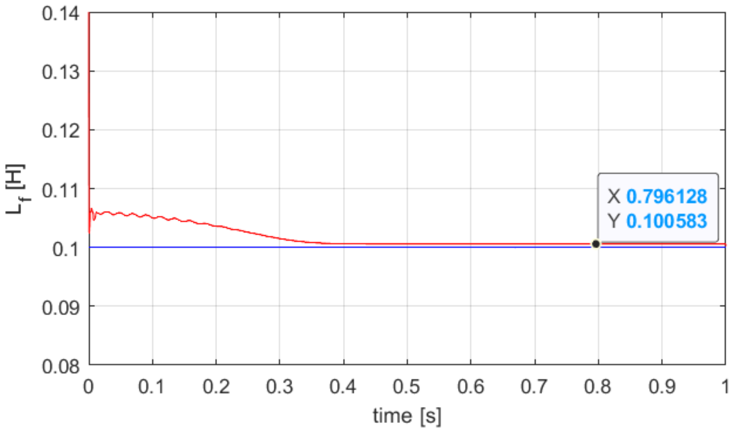

Fig.8. The variation over time of the estimated L_f inductance, together with the reference value.

From the previous figures, it can be observed that the on-line estimation method converges to the reference values in approximately 0.4 [s] (identical to the starting time motor), for the case in which the three-phase supply voltages have a frequency of 50 [Hz].

From Fig. 7, it can be observed that the steady-state error in the on-line estimation of the resistance R_f is approximately 0.45 [Ω].

On the other hand, from Fig. 8, it can be observed that the steady-state error in the estimated inductance L_f is approximately $5.8 \cdot 10^{-4}$ [H].

5. CONCLUSIONS

The article presents a new method for the online estimation of the core-loss resistance R_f and the eddy-current-associated leakage inductance L_f of an induction motor. Simulation results confirm that the proposed online estimation method for R_f and L_f is both accurate and robust, demonstrating its potential for practical implementation in industrial and research applications.

The detailed mathematical formulations of the proposed estimators, together with the results obtained from the Matlab-Simulink simulations, offer substantial support for researchers and practitioners working with induction motors.

A promising direction for future work is the practical implementation of the proposed estimator.

REFERENCES

[1]. Aissa K., Eddine K.D., *Vector control using Series Iron Loss Model of Induction Motors and Power Loss Minimization,* World Academy of Science, Engineering and Technology, Vol.52, pp. 142-148, 2009.

[2]. Boldea I., Nasar S.A., *Vector Control of AC Drives,* CRC Press, 1992.

[3]. Boldea I., Nasar S.A., *Unified Treatment of Core Losses and Saturation in the Orthogonal-Axis Model of Electric Machines,* IEE Proceedings B, Electric Power Applications, Vol.134, No. 6, pp.355-363, 1987.

[4]. Levi E., *Iron core loss effects in indirect rotor flux oriented induction machines*, Proceedings of Melecon'94, Mediterranean Electrotechnical Conference, Vol.2, pp. 766-769, 1994.

[5]. Levi E., *Impact of Iron Loss on Behavior of Vector Controlled Induction Machines,* IEEE Transactions on Industry Applications, vol.31, pp.1287-1296, 1995.

[6]. Pana T., Stoicuta O., *Stability of the Vector Drive Systems with Induction Motors*, Mediamira Publishers, 2016.

[7]. Stoicuta N., Stoicuta O., Pana T., *On-line Electrical Parameters Estimation of an Induction Motor*, International Conference on Applied and Theoretical Electricity (ICATE), pp. 1-4, Craiova, 2018.

[8]. Stoicuta O., *The utilization of the S-Function block in simulation of the mathematical model of induction motor with iron loss*, Annals of the University of Petrosani, Electrical Engineering, Vol.18, pp. 5-20, 2016.

[9]. Stoicuta O., *Mathematical modeling and simulation of induction motor with eddy current leakage inductance in the core loss branch*, Annals of the University of Petrosani, Electrical Engineering, Vol.24, pp. 117-128, 2022.

[10]. Vas P., *Vector Control of AC Machines*, CRC Oxford, 1990.